The experimental upper limit for $\sin\Phi$ is 0.26 (Ref. 2).

Putting this value in (18), we find as a typical maximum value for γ

$$(\gamma_{K=0})_{\max}\approx 1.3\times 10^{-7}. \qquad (20)$$

In the case of the neutron we have to consider Figs. 2(a) and 2(b) for the evaluation of the electric-dipole moment. However, we find that the contribution to the $\ln\xi$ term from Fig. 2(b) vanishes. Thus we expect the electric-dipole moments of the neutron and proton to be of the same order of magnitude.

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Longitudinal Behavior of Electromagnetic Showers*

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The need to establish recognition patterns for high-energy photons and electrons in cosmic-ray work, and more recently, around high-energy accelerators, has stimulated theoretical and experimental investigations into the properties of electromagnetic cascades. The most recent results of statistical computations, for longitudinal development and for lateral and angular spread, are in large part inconsistent with earlier published data. In order to study the longitudinal behavior of electron-induced showers, measurements have been made with a monoenergetic electron beam (energies 100–1000 MeV) at the 1.5-BeV CIT electron synchrotron. Buildup and energy dissipation were investigated, through the sampling of showers generated in lead of variable thicknesses, by means of a Lucite Čerenkov counter. Average numbers of shower particles with energies above 10 MeV are given for these incoming energies and penetration depths up to 10 radiation lengths; also, shower fluctuations are presented for the same points. The results of this experiment can readily be compared with the data recently computed by Crawford and Messel. Agreement appears to be satisfactory.

INTRODUCTION

THE buildup and decay of electromagnetic cascade showers, initiated by high-energy photons or electrons, has long been studied in connection with cosmic-ray work. More recently, the cascading properties have been used in increasing measure for the selective detection of showering particles in work around high-energy particle accelerators.

Although the equations describing the fundamental processes involved in buildup and decay of showers are well established, analytical solutions of the shower distributions taking into account the full amount of physical phenomena are impossible to obtain. Various approaches and approximations have been put forward, notably separating the problems of longitudinal, lateral, and angular structure. More recently, calculations using statistical models have been made to obtain numerical data for various sets of input parameters. In this approach, it is easier to take all of the important cross-section data into account and not to introduce too many oversimplifications. The most recent data of Messel *et al.*[1] and Crawford and Messel[2] differ appreciably from earlier results obtained by Wilson[3] for the longitudinal development of showers, as well as from approximate analytical solutions as presented by Belenkii and Ivanenko[4] and others.[5] There has been no experimental check so far on the validity of Messel's data. Earlier work, e.g., by Lal and Subramanian,[6] has too many inherent uncertainties to allow for a quantitative confirmation.

The structure of electromagnetic showers (longitudinal, lateral, and angular distributions) is of vital interest for the identification of particles in heavy backgrounds, either singling out high-energy photons and electrons in the presence of copious π, K, $p\cdots$production from targets around high-energy accelerators, or discriminating against them; and contingent knowledge may help to establish recognition patterns for incoming photons and/or electrons of given energies. Therefore,

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[2] D. F. Crawford and H. Messel, Phys. Rev. **128**, 2352 (1962).

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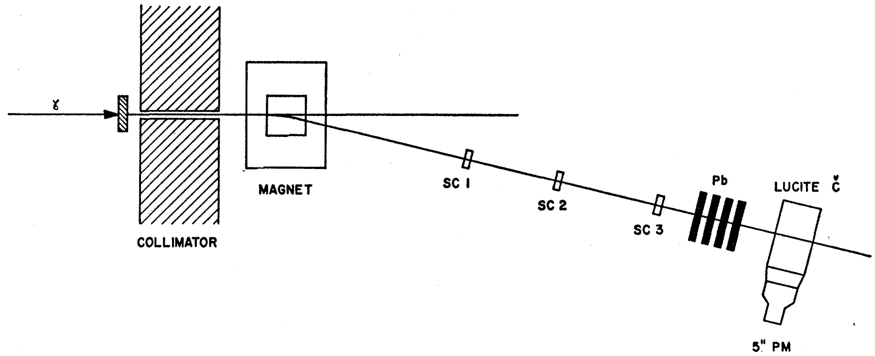

FIG. 1. Experimental setup. Size of Lucite Čerenkov radiator: 5 in.×9 in.×3 in. Area of defining counter S_3: 1 in.×1 in.

it appears desirable to acquire experimental data on shower buildup and decay in heavy media suitable for work in this field. It was our purpose, in the work reported here, to look into the behavior of electromagnetic showers in the energy region of interest for work around our 1.5-BeV synchrotron, and investigate their longitudinal structure at closely-spaced energy intervals; and to compare our results with the computations of the Australian group, who have been turning out data for a number of varying parameters.[1,2,7] Agreement should establish confidence in the approximations made in their computations.

Although our method, with slight modifications, could be expanded to take data on the lateral spread of the showers, we have not done work in this field. Our method, further, does not allow for investigations of problems of shower correlations.

SUITABLE METHODS FOR EXPERIMENTAL CHECKS

For shower investigations, all track chambers could be used and have indeed been recently used (bubble chamber,[8] cloud chamber,[6] spark chamber[9]) in addition to emulsion stacks as used in cosmic-ray work. They offer the inherent difficulty that calibrations are usually difficult to perform; that analysis of the data is tedious, and that low-energy cutoffs are difficult to establish, so that comparison with computed values cannot easily be effected. The last argument also holds for configurations involving scintillator counters; very-low-energy electrons can generate large pulses in the scintillator. It appears reasonably straightforward to use Čerenkov radiators of a thickness corresponding to a given cutoff energy; i.e., an electron needs an incident energy of at least E_{cutoff} in order to traverse the full radiator, and thereby generate a full-sized pulse. In this experiment, a Lucite radiator was used, which has the additional advantage of offering excellent light transmittance properties. Lastly, evaluation of data collected by this method is quick and relatively simple.

EXPERIMENTAL SETUP

The experimental setup is schematically shown in Fig. 1. A small fraction of a bremsstrahlung γ beam from the 1.5-BeV CIT electron synchrotron is converted, and subsequent momentum separation is effected in a magnet. Energy calibration of the ensuing monoenergetic electron beam, defined by counters S_1, S_2, S_3, is correct to $\pm 3\%$. In our configuration, it was possible to vary electron energies from 0.1 to 1.0 BeV. The experiment was done only with lead as a converter; appropriate thicknesses of lead were packed in front of a Lucite radiator of ≈ 7.5 cm thickness. The Čerenkov light emitted by the electrons in the Lucite was collected onto the cathode of a 5-in. phototube (RCA 7046). The electronic schematic is fairly conventional. The signal, with proper delay, passes through a 60-nsec linear gate into a linear integrator-amplifier circuit and is subsequently analyzed and stored in a 256-channel pulse-height analyzer. The gate is opened by a triple coincidence from the three defining counters S_1, S_2, S_3. Since

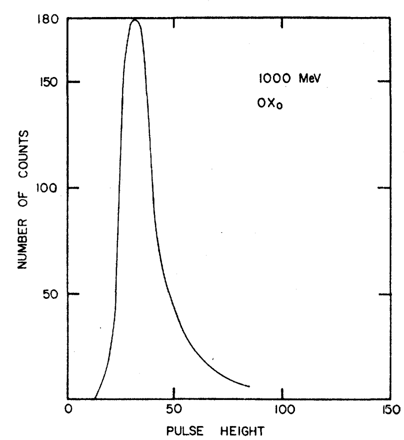

FIG. 2. "Check run": Pulse-height distribution generated by 10 000 $\beta \cong 1$ electrons incident on the Lucite radiator. Tail is due to the buildup of showers within Lucite (thickness $\approx 1/7 X_0$), and to high-energy knock-on electrons.

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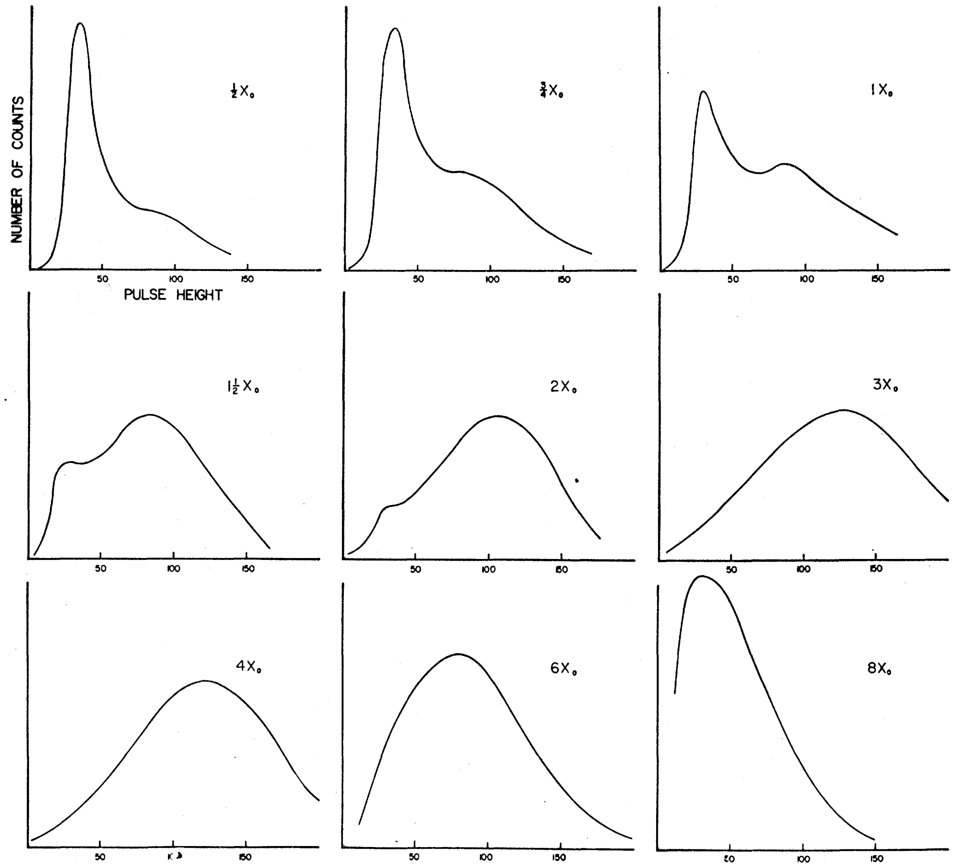

FIG. 3. Buildup and decay of electron-initiated showers: differential spectra for various penetration depths. E_0=1000 MeV. Material: lead ($E \geq 10$ MeV).

TABLE I. Average number of electrons at given shower penetration depth in lead for given energy of incident electron generating shower (1st line); standard deviation for same (2nd line). Data not reduced for slight influences of angular opening of shower and of below-cutoff electrons.

$t(X_0)$ \ E_0(MeV)		100	200	300	400	500	600	700	800	900	1000
0.5	$\langle N\rangle$		1.13	1.19	1.25	1.34	1.37	1.37	1.39	1.42	1.46
	σ		0.41	0.44	0.47	0.57	0.56	0.59	0.59	0.60	0.65
1.0	$\langle N\rangle$	1.07	1.30	1.64	1.85	1.99	2.14	2.21	2.33	2.38	2.54
	σ	0.39	0.55	0.80	0.97	1.06	1.11	1.15	1.26	1.28	1.34
2.0	$\langle N\rangle$	0.86	1.38	1.92	2.38	2.79	3.05	3.31	3.58	3.72	4.08
	σ	0.38	0.57	0.92	1.15	1.29	1.40	1.49	1.61	1.65	1.81
2.5	$\langle N\rangle$		1.34	1.87	2.33	2.84	3.25	3.61	3.83	4.18	4.71
	σ		0.56	0.93	1.14	1.36	1.46	1.57	1.73	1.79	1.94
3.0	$\langle N\rangle$	0.62	1.23	1.76	2.30	2.84	3.24	3.66	4.02	4.30	4.78
	σ	0.33	0.59	0.90	1.08	1.23	1.36	1.46	1.58	1.63	1.79
3.5	$\langle N\rangle$		1.09	1.52	2.05	2.57	3.17	3.64	4.02	4.48	4.90
	σ		0.58	0.84	1.05	1.22	1.41	1.52	1.68	1.70	1.79
4.0	$\langle N\rangle$		0.82	1.34	1.92	2.46	2.90	3.36	3.85	4.21	4.74
	σ		0.51	0.78	0.99	1.17	1.25	1.35	1.49	1.55	1.64
5.0	$\langle N\rangle$		0.64		1.51	1.93	2.35	2.86	3.30	3.72	4.17
	σ		0.46		0.85	0.99	1.12	1.23	1.32	1.42	1.50
6.0	$\langle N\rangle$		0.54	0.79	1.17	1.54	1.85	2.23	2.63	3.04	3.46
	σ		0.41	0.56	0.74	0.87	1.01	1.11	1.19	1.32	1.41
7.0	$\langle N\rangle$			0.65	0.90	1.13	1.46	1.76	2.04	2.36	2.77
	σ			0.50	0.62	0.74	0.86	0.99	1.08	1.19	1.28
8.0	$\langle N\rangle$				0.67	0.93	1.22	1.39	1.62	1.88	2.20
	σ				0.54	0.65	0.76	0.87	0.97	1.07	1.14
9.0	$\langle N\rangle$							1.04	1.19	1.31	1.56
	σ							0.66	0.70	0.70	0.81
10.0	$\langle N\rangle$									1.07	1.24
	σ									0.66	0.74

the quality of our results depends vitally on the accuracy and constancy of the pulse-height information, and since, for measurements on the tail end of the shower, small pulses had to be analyzed, great emphasis had to be put on the linearity of the circuitry, on the absence of an electronic pedestal for the analyzer, and on a reliable calibration procedure. This was done (1) by running electronic calibrations by means of feeding standard simulated phototube pulses from a pulse generator through the electronics system; (2) by repeatedly removing all the shower buildup material from the radiator so that only the standard distribution of "noninteracting" electrons would show up. Then, check runs were taken as points of reference. This procedure corrects for any long-time drifts in phototube gain and possible errors in high-voltage settings. In this manner, it was possible to reproduce our data reliably within statistical limits.

RESULTS

Data were taken at energies of incoming electrons E_0, from 100 through 1000 MeV, in steps of 100 MeV; at each of these energies, pulse-height distributions were measured for shower penetration depths of 0, $\frac{1}{2}$ radiation length X_0, $1X_0$, $\cdots$ advancing by $\frac{1}{2}X_0$ until well beyond the shower maximum, then at intervals of $1X_0$ each. At each position and energy, 10 000 counts were accumulated. The standard distribution of one electron of $\beta\approx 1$ is shown in Fig. 2. The mean value is defined as unity (apparent number of electrons $\langle N\rangle = 1$). It can be seen that the front rise is close to Gaussian, but the back slope tails out. The front rise is mainly determined by photoelectron statistics at the photocathode (plus, to a lesser degree, secondary-electron statistics at the first dynodes); the tail is due to showers building up in the lucite radiator (thickness $\approx\frac{1}{7}X_0$), and to high-energy knock-on electrons.

The gradual buildup of the shower can be followed in Fig. 3, where we show the pulse-height distributions generated while sampling the shower at penetration depths, in lead, of $\frac{1}{2}X_0$, $\frac{3}{4}X_0$, $1X_0$, etc. In the front section of the shower, the most probable process is passage of an electron without interaction or with production of one or several bremsstrahlung quanta without conversion of the latter. A secondary peak can be seen to build up at a pulse height three times the first, indicating that the next most probable fate of an incoming electron is to undergo a bremsstrahlung process with subsequent creation of an electron-positron pair. A two-electron peak is, for incident electrons, understandably absent, but would prevail for incoming photons. With increasing penetration depth, the "noninteracting" electron peak levels off, the three-electron peak gains weight and broadness, until at the shower maximum the shower fluctuations have completely smeared out any structure. Here, the distribution is fairly symmetrical. At penetration depths beyond $4X_0$, we move visibly into the tail of the shower; the distributions become wide, the mean values lower. As the weight of the low-energy particles increases, the maximum of the distribution shifts below the position of the peak due to passing $\beta\approx 1$ particles.

Similar distributions, from each of the energy settings mentioned, were fed into an IBM-7090 computer which determined the mean values, corrected for possible calibration errors, and compared these mean values with the one defined as unity, "noninteracting" electrons. By this procedure, which yielded reproducible data from runs taken over an appreciable time span, the values were obtained for average numbers of electrons at given penetration depths of showers ranging, in

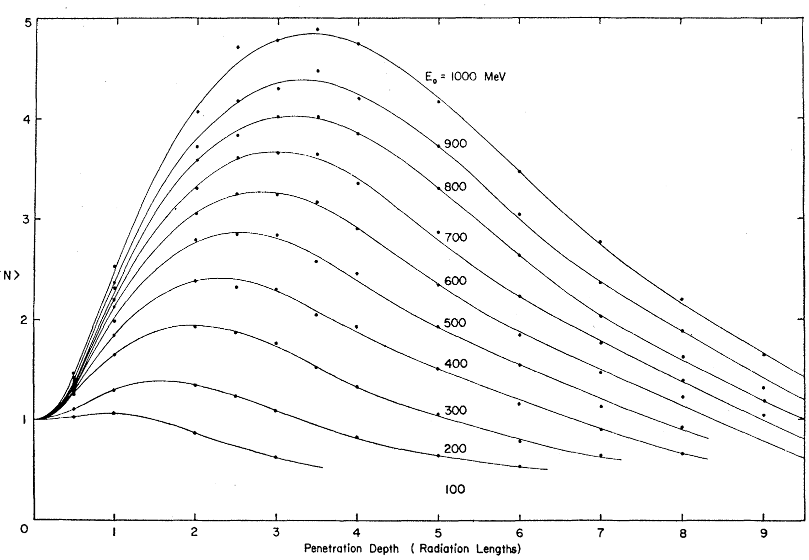

FIG. 4. Average numbers of electrons ($E\gtrsim 10$ MeV) at given penetration depths of shower, for incoming electron energies ranging from 100 to 1000 MeV. Data not reduced for below-cutoff electrons, angular opening.

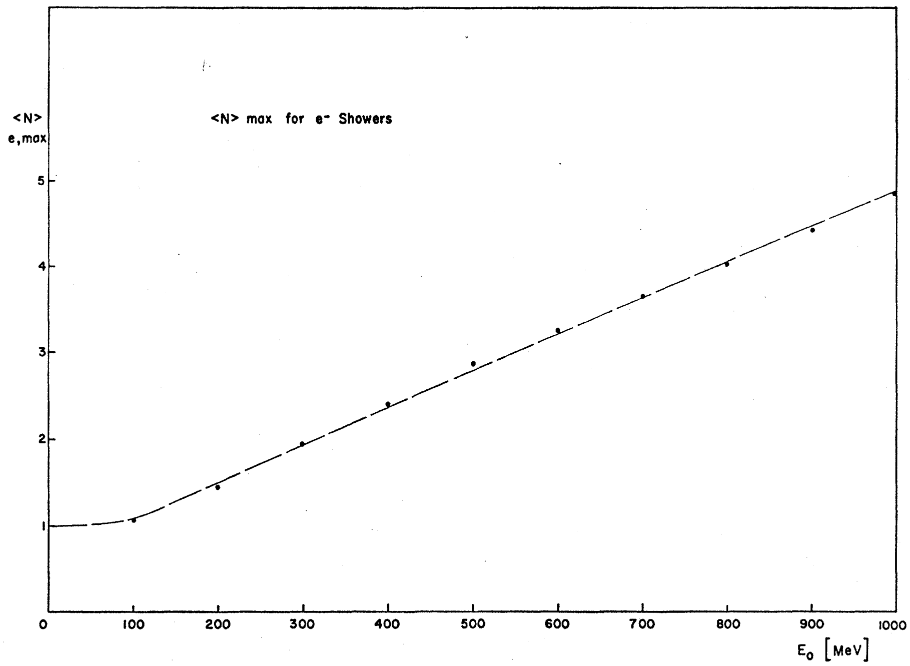

FIG. 5. Average number of electrons ($E \gtrsim 10$ MeV) at point of shower maximum, as a function of energy of incoming electrons.

incoming energies, from 100 to 1000 MeV. These numbers are given in Table I, and displayed in Fig. 4. They constitute our final results on the average longitudinal behavior of electron-induced showers, with the average number of particles defined as the ratio of the mean value of any given pulse-height distribution to the mean of the unit distribution. A possible systematic error may arise from the energy calibration of the monoenergetic beam. The slight scatter of the experimental points about the smooth curves (eye-fits) is caused by drifts in the electronics. The statistical uncertainty is $\pm 1\%$. Also given in Table I, together with the average number of shower particles at given depths, are the respective shower fluctuations. They were computed from the second moment of our experimental pulse-height distributions, with proper correction for the inherent width of the reference ("unit") distribution. The numbers of particles at the shower maximum, as a function of incoming electron energy, are displayed in Fig. 5. They are seen to rise close to linearly above $E_0 = 100$ MeV. Approximately, they follow

$$\langle N\rangle_{\max}|_{E>10\ \mathrm{MeV}} = 4.1E_0 + 0.65 (E_0 \text{ in BeV}).$$

DISCUSSION—COMPARISON WITH COMPUTED VALUES

It should be noted that our experimental method puts a cutoff at approximately $E_0 = 10$ MeV. Therefore, a fairly straightforward comparison with the data obtained by Crawford and Messel[2] can be made. Their Monte Carlo computations take into account multiple scattering, collision, Compton effect, and the Bethe-Heitler cross sections for pair production and bremsstrahlung, accurate at both high and low energies. In their work, they "lose" electrons and photons from the shower, i.e., they disregard their further fate, once they fall below a 10-MeV energy cutoff, or once they undergo a back scattering at $E > 10$ MeV.

In order to compare our results with their computed values, we have to modify our data to take account of the following effects (the comparison of unmodified experimental and theoretical values, for energies given in Ref. 2, is displayed in Fig. 6) :

(1) Our experiment "sees" electrons, although with reduced weight, below the cutoff energy of 10 MeV.

(2) Electrons traveling at an angle to the shower axis will have a longer path in the radiator than axial particles, will therefore generate higher-than-unit pulses.

(3) Photons of energy <10 MeV may still convert or undergo Compton collisions, and yield electrons of $\beta \approx 1$, but of small penetration depth, generating small pulses.

(4) Shower leakage out of the sides of the radiator may occur for particles traveling at large angles to the axis.

The effect of (4) has been estimated, for our geometries, and found to be negligible. In order to estimate the amount of Čerenkov light generated by low-energy ($E < 10$ MeV) electrons (1), we used the numbers of electrons lost because of cutoff requirements, in given penetration intervals, as mentioned in Ref. 2; taking their mean energy to be ≈ 7 MeV, we averaged over lead thicknesses to be traversed and angular distribution, and computed the average fraction of the unit pulse height generated in the Lucite by these below-cutoff particles.

Reduction of the influence of the shower's angular spread, (2), on our data was effected by deducing a suitable average angular opening of the shower from Ref. 1, calculated with the same Monte Carlo program as Ref. 2, and by taking the ensuing prolonged path in the radiator into account. The only published data on angular opening are for an incoming energy of $E_0 = 1000$ MeV. Therefore, a quantitative comparison could be drawn for this case only.

As far as (3) is concerned, no reliable estimate could be obtained. On one hand, according to Ref. 2, the average energy of photons lost to the shower on account of the 10-MeV cutoff is 0.91 MeV, i.e., too low to generate Compton or pair electrons which might produce appreciable pulses in the Lucite radiator. On the other hand, the average number of photons lost to the showers, per unit penetration depth, is much higher than the number of electrons lost in the same interval. It is possible, therefore, that an appreciable number of photons of energies between 1 and 10 MeV have a chance to reconvert or undergo Compton scattering. This could become noticeable at larger penetration depths, because there the large mean free path of photons of the "hole region" (i.e., at the minimum of the total cross section, ≈ 2.5 MeV in Pb) could lead to an accumulation of these photons. However, they should be more or less isotropically distributed. In the absence of any reliable data, we did not reduce our results for their possible influence.

Figure 7 shows a comparison of our results, reduced in the fashion described above, with the results of Crawford and Messel,[2] for an incident electron energy of 1000 MeV. It can be seen that agreement is satisfactory over the full range of the curves displayed, and excellent in the range of most interest to the experimentalist wishing to establish a recognition pattern, i.e., around the maximum. The error flags in our points are an estimated uncertainty on account of statistics, and of drifts in the electronics system. The statistical uncertainty in the computed data of Ref. 2 is $\approx \pm 4.5\%$. It should be mentioned that there is no adjustable parameter in the reduction of our data for this comparison.

The discrepancy in the tail of the shower, which yields an experimental decay constant of $0.26(X_0)^{-1}$ for our measurements versus $0.33(X_1)^{-1}$ for Messel's computations, can be due to the presence of a large number of low-energy γ's in that part of the shower, thus effectively reducing the cutoff of the experiment; or to the fact that the computed values of Ref. 2 become inherently less reliable as they move into the shower tail.

A further point of agreement between our data and the calculated distribution is gained from extrapolating the exponential decay of the shower beyond the measured values. This gives a measure for the energy remaining in the shower, if we assume that the number of electrons above effective cutoff is proportional to the total energy of the shower in the region of the shower tail. This is roughly the case, but should yield slightly too high a value. (Actually, the average energy carried by an electron with $E > 10$ MeV decreases slowly as we move farther out into the tail; cf. Ref. 2.) Our extrapolation shows 11.7% of the 1-BeV shower leaking out beyond a penetration depth of 10 X_0; the corresponding value of Crawford and Messel[2] is slightly over 10%.

The data displayed in Fig. 4 and in Table I also allow for a determination of total track lengths. They are approximately proportional to the energies of incident particles according to

$$T[X_0] \approx 0.035 E_0 \quad (E_0 \text{ in MeV}).$$

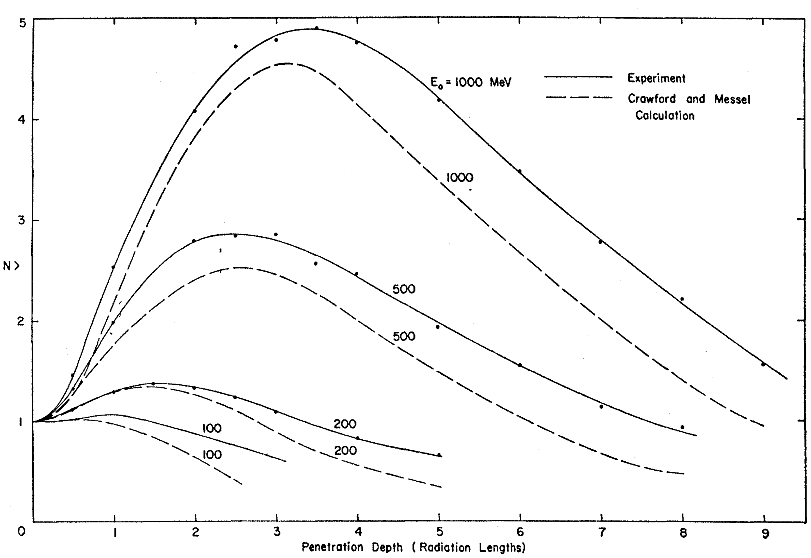

FIG. 6. Average numbers of electrons at given penetration depths, not reduced. Comparison with values calculated by Crawford and Messel (Ref. 2) (dashed curve).

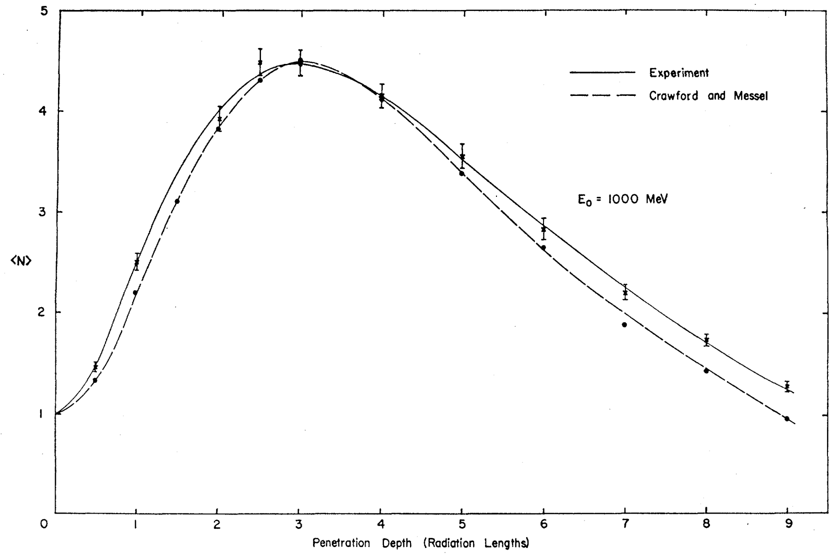

FIG. 7. Average number of electrons at given penetration depths of 1-BeV electron-initiated shower in Pb. Experimental data reduced for influence of below-cutoff electrons and angular opening of shower. Comparison with values calculated by Crawford and Messel (Ref. 2).

This proportionality is, of course, the relation at the basis of energy determination by means of total absorption Čerenkov counters.

The correlations in individual showers make the measurement of particle energies by means of the determination of $\langle N\rangle_{\max}$ considerably less reliable, as shown by the fluctuations reported in Table I. Averaging over many events of one energy, however, the accuracies obtainable in this manner will be greatly enhanced (cf. error flags in Fig. 7).

CONCLUSIONS

The average longitudinal behavior of electron-induced electromagnetic showers in the energy range between 100 MeV and 1 BeV was investigated by means of a Lucite Čerenkov counter behind appropriate thicknesses of lead. Average numbers of electrons at given penetration depths were determined in steps of 100 MeV, for shower penetration depths up to 10 radiation lengths. Likewise, the fluctuations in the longitudinal structure of the shower were computed from the experimental distributions. All the data obtained are compatible with an exponential decay of the cascades at large depths according to average number of shower particles with $E>10$ MeV, $\langle N\rangle\sim e^{-\lambda x}$ with $\lambda=0.26(X_0)^{-1}$, so that extrapolation beyond the measured values is possible.

For several of the energies measured, recent Monte Carlo calculations of Messel *et al.*[1,2] offer computed values for fairly straightforward comparison. After proper matching of experimental conditions and theoretical approximations, a quantitative comparison at $E_0=1000$ MeV shows satisfactory agreement (with a possible small discrepancy in the region of the shower tail), while results of previous statistical[3] and analytical computations show large deviations.

It is hoped that an unfolding of the experimental distribution reported here will likewise yield the differential probabilities of finding 0, 1, 2, $\cdots$ electrons at penetration depth $t(X_0)$, for the incident energy range covered, and that these data will be presented in the near future.

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